Spatial Analysis of Regional Climate Experiments: Functional ANOVA and Heat Stress

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Outline

• Comparing winter precipitation.
  – NARCCAP NCEP-driven runs.

• A preliminary study of heat stress.
  – NARCCAP GFDL-driven timeslice/regional climate model.
  – Two-factor functional analysis of variance.
NARCCAP

- North American Regional Climate Change Assessment Program (www.narccap.ucar.edu)
  - NCAR, ISU, CCCma, OURANOS, LLNL, GFDL, Hadley, Scripps, PNNL, USSC, etc.
  - NSF, NOAA, DOE, EPA

- Systematically investigate the uncertainties in regional scale projections of future climate and produce high resolution climate change projections using multiple RCM and multiple GCM simulations.

- 4 GCMs provide boundary conditions for 6 RCMs
  - balanced half-fraction
NCEP Experiment

- Six regional models
  - CRCM (OURANOS/UQAM), ECPC (UC San Diego/Scripps), HRM3 (Hadley Centre), MM5I (Iowa State U.), RCM3 (UC Santa Cruz), WRFP (PNNL)

- Boundary conditions supplied by NCEP Reanalysis II.

- 1981 – 2000 (20 years)

- Average daily precipitation (mm) – winter (DJF)

- Interpolated to a common grid: $120 \times 98 = 11,760$ grid boxes
Analysis of Variance

\[ Y_{ij} = \mu + \alpha_i + \epsilon_{ij} \]

- For every grid box (this grid-box is in eastern Nebraska):
  - \( Y_{ij} \) is the response (transformed precipitation) for the \( i \)th model and the \( j \)th year.
  - \( \mu \) is a common mean
  - \( \alpha_i \) is a RCM-specific effect
  - \( \epsilon_{ij} \) is the error or residual
Analysis of Variance

\[ Y_{ij} = \mu + \alpha_i + \epsilon_{ij} \]

- Testing the null hypothesis \( H_0 : \alpha_1 = \ldots = \alpha_6 = 0 \):

<table>
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<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p-value</th>
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<tr>
<td>RCM</td>
<td>5</td>
<td>0.163</td>
<td>0.0326</td>
<td>15.3</td>
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<td>Residual</td>
<td>114</td>
<td>0.243</td>
<td>0.00213</td>
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- Conclusion: strong evidence of differences in the RCM means.
Map of pointwise p-values: strong evidence of differences in RCM means over nearly every grid box in the domain ???
Problem: correlated residuals at neighboring grid-boxes.

Result: invalid inference – any conclusions based on the p-value map are suspect.
The goal is to partition the variation into specific effects:

- $Y_{ij}$ is the vector response (transformed precipitation) for the $i$th model and $j$th year.
- $\mu$ is the vector mean common to all RCMs
- $\alpha_i$ is the vector RCM-specific effect
- $\epsilon_{ij}$ is the vector residual.
Functional Analysis of Variance

Y_{ij} = \mu + \alpha_i + \epsilon_{ij}

- The innovation is that each of these effects is a *surface*.
- Each effect is considered a realization from a random process.
- Gaussian fields are often used as prior distributions; inferences about the effects involve conditioning on the observed output fields.
Pointwise probabilities that the model-to-model variation is larger than the year-to-year variation (analogous to small p-values in a traditional ANOVA).
A Statistical Model

- A common approach involves a three-level hierarchy:
  
  Data model: \([data|process, parameters]\)
  
  Process model: \([process|parameters]\)
  
  Prior model: \([parameters]\)

- Simplifies the problem by factoring a complicated distribution into a series of conditional distributions.

- Inference involves sampling the posterior distribution:
  
  \([process, parameters|data] \propto [data|process, parameters][process|parameters][parameters]\)
A Statistical Model

- A hierarchical structure:

  **Data model:** \( Y_{ij} \sim \mathcal{N} \left( \mu_i, \sigma_i^2 V(\phi_i) \right), \ i = 1, \ldots, 6, j = 1, \ldots, 20 \)

  **Process model:** \( \mu_i \sim \mathcal{N} \left( \mu, \sigma^2 V(\phi) \right) \)

  **Prior model:** \( \mu \sim \mathcal{N} \left( \text{NCEP}, \sigma^2 \mu V(\phi_\mu) \right) \)

- \( \{Y_{ij}\} \) are (transformed) daily average precipitation fields
- \( \{\mu_i\} \) are model specific means; \( \mu \) is the “grand” mean
- \( \{\sigma_i^2\}, \sigma^2, \sigma^2_\mu \) are scale parameters
- \( \{\phi_i\}, \phi, \phi_\mu \) are spatial dependence parameters

- Prior distributions on scale and spatial dependence parameters are non-informative.
A Statistical Model

- An alternative (ANOVA) formulation:

\[ Y_{ij} = \text{NCEP} + \eta + \alpha_i + \epsilon_{ij}, \quad i = 1, \ldots, 6, j = 1, \ldots, 20 \]

- \( \eta \) is a common component to all fields and explains variation beyond NCEP.
  - \( \mu = \text{NCEP} + \eta \).
  - \( \{\alpha_i\} \) are RCM-specific components.
    - \( \mu_i = \text{NCEP} + \eta + \alpha_i \).
  - \( \{\epsilon_{ij}\} \) represent year-to-year variation.

- Sain, Kaufman, and Tebaldi (2009, in preparation)
NCEP Experiment

- Recall the functional ANOVA model:

\[ Y_{ij} = \text{NCEP} + \eta + \alpha_i + \epsilon_{ij}, \quad i = 1, \ldots, 6, \quad j = 1, \ldots, 20 \]

- Compare \( \mu \) to NCEP – how do the RCMs on average compare to the driving model?

- Compare \( \{\alpha_i\} \) – how consistent are the RCMs and how do they compare with each other?

- By drawing samples from the posterior (ensemble), we can address these questions giving insight to the sources of uncertainty in the collection of RCM output.
• Difference between posterior mean for $\mu$ and the mean NCEP.

• Average daily winter precipitation (transformed).
Pointwise probabilities that draws from the posterior distribution of $\mu$ are greater than the mean NCEP field. Red (credibly true); white (credibly false).
Pointwise probabilities that the model-to-model variation is larger than the year-to-year variation.
• Diagonal elements \( \hat{P}[\alpha_i > 0] \)

• Off-diagonal elements \( \hat{P}[\alpha_i > \alpha_j] \)

• Red (credibly true); Blue (credibly false)
Heat Stress: A Preliminary Study

- Two types of dynamic downscaling: a GFDL time-slice and a GFDL-driven RCM (RCM3; UC Santa Cruz).
  - Geophysical Fluid Dynamics Laboratory (GFDL; NOAA)

- Both timeslice and RCM use the A2 scenario.

- Current (1971-2000) and future (2041-2070) runs.

- Focus on summer (May-September) heat stress.
  - Output interpolated to a common grid (134 × 83).

- Examine differences in the two models as well as changing heat stress in North America.
What is a Heat Wave/Heat Stress?

- “...an extended period of unusually high atmosphere-related heat stress, which causes temporary modifications in lifestyle, and which may have adverse health consequences for the affected population.”
  - Intensity and duration and local climatology.

- We adopt the definition of heat stress put forth by Meehl and Tebaldi (2004) in their study of global climate models: maximum of the 3-day running mean of the overnight minimum temperature.
  - Captures persistence and (lack of) overnight cooling.
Heat Stress (Timeslice)
Heat Stress (RCM)

1980

1990

2000

2050

2060

2070
Heat Stress (Means)

Current

Future

Difference
A Functional ANOVA Model

- Let $Y_{ijt}$ denote the $i$th model (timeslice vs RCM; $i = 0, 1$), the $j$th run (current vs future; $j = 0, 1$), at the $t$th time ($t = 1, \ldots, 30$):

$$Y_{ijt} = \alpha_0 + i\alpha_1 + j\alpha_2 + ij\alpha_3 + \epsilon_{ijt}$$

- Assume each component is generated from a Markov random field:

$$\alpha_0 \sim \mathcal{N} (\mu_{\text{curr}}, \Sigma(\theta_0)) \quad \alpha_1 \sim \mathcal{N} (0, \Sigma(\theta_1))$$

$$\alpha_2 \sim \mathcal{N} (\mu_{\text{diff}}, \Sigma(\theta_2)) \quad \alpha_3 \sim \mathcal{N} (0, \Sigma(\theta_3))$$

- $\mu_{\text{curr}}$ and $\mu_{\text{diff}}$ are average fields of the current and difference in heat stress computed from the driving global GFDL model.
A Functional ANOVA Model

- The error term, \( \epsilon_{ijt} \) is broken up into two pieces:

\[
\epsilon_{ijt} = \gamma_j (t - 15.5) + \eta_t
\]

where

\[
\gamma_j \sim \mathcal{N} \left( \gamma_j^*, \sigma_\gamma^2 \right) \quad \eta_t \sim \mathcal{N} \left( 0, \Sigma(\theta_t) \right)
\]

- \( \gamma_j^* \) are average slopes from the control and future runs of the driving global GFDL model.
100 draws from the posterior of $\gamma_0$ (left) and $\gamma_1$ (right).
Posterior Means

- $\bar{\alpha}_0$ represents current timeslice.
- $\bar{\alpha}_1$ adjusts for current RCM.
- $\bar{\alpha}_2$ adjusts for future run.
- $\bar{\alpha}_3$ is an interaction.
Posterior Means

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A Quick Look at Temperatures

\[ \bar{\alpha}_0 \]
\[ \bar{\alpha}_1 \]
\[ \bar{\alpha}_2 \]
\[ \bar{\alpha}_3 \]

\[ \hat{P}[\alpha_0 > \mu_{curr}] \]
\[ \hat{P}[\alpha_1 > 0] \]
\[ \hat{P}[\alpha_2 > \mu_{diff}] \]
\[ \hat{P}[\alpha_3 > 0] \]

JJA Ave Temp – \( p < 0.05 \) – blue; \( p > 0.95 \) – red.
- A single draw from the posterior for $\alpha_2$.
- Contour represents an increase in heat stress by 3.0 degrees.
- Posterior mean for $\alpha_2$.
- Contour lines represent an increase in heat stress by 3.0 degrees for 20 randomly sampled draws from the posterior of $\alpha_2$. 
• Pointwise posterior probability that $\alpha_2(s) > 3.0$.

• Regions where all draws are greater than 3.0 (inside wide contour) or where no draws were greater than 3.0 (outside thin contour).
Varying thresholds ($\tau = 2.0, 3.0, 4.0$) for timeslice ($\alpha_2$; top) and RCM ($\alpha_2 + \alpha_3$; bottom).
Pointwise probability for change greater than 3.0 for both models.
Questions?

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Thank You!