Spatial Analysis of Regional Climate Experiments: Functional ANOVA and Heat Stress

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Outline

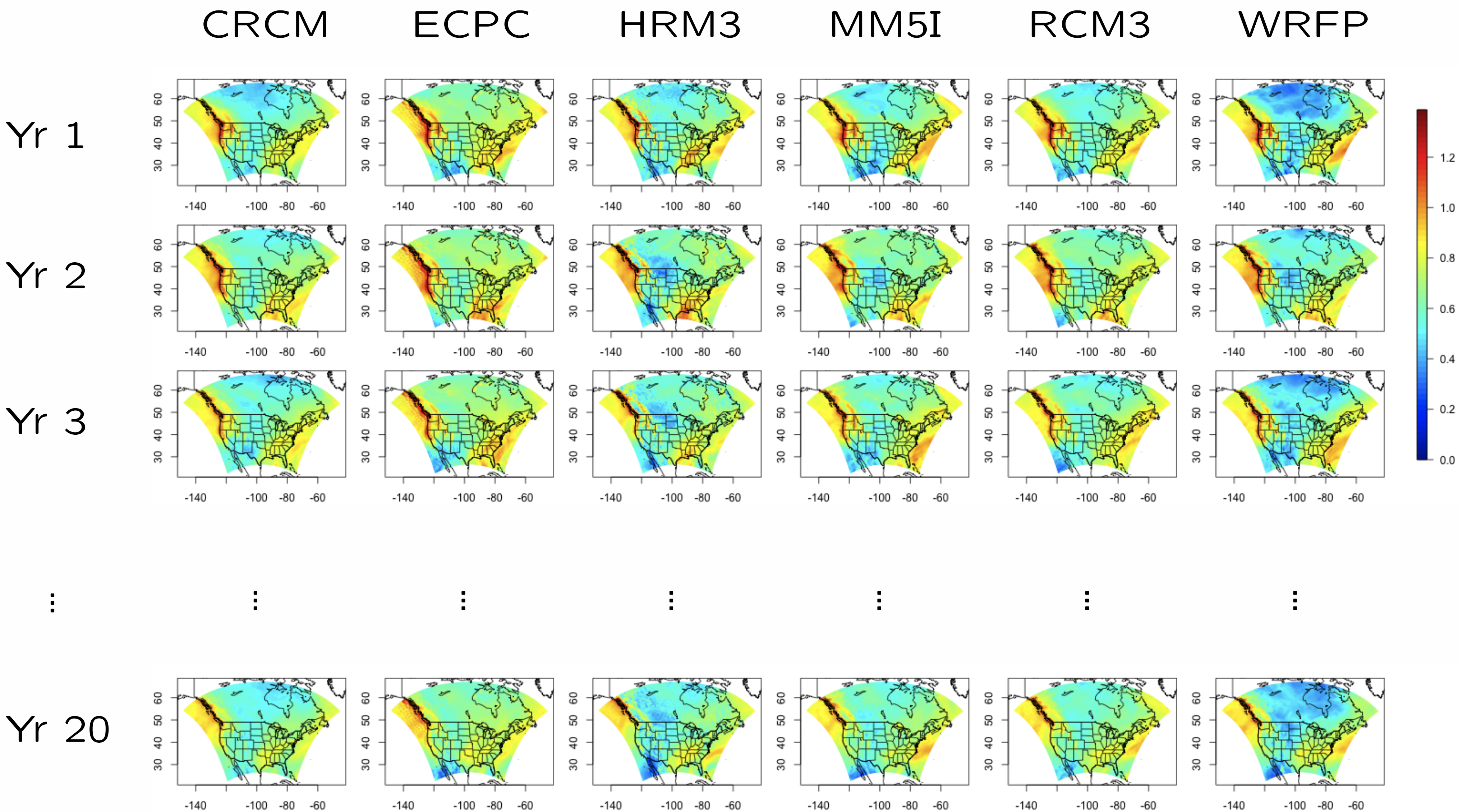
- Comparing winter precipitation.
  - NARCCAP NCEP-driven runs.
  - Single-factor functional analysis of variance.
- A preliminary study of heat stress.
  - NARCCAP GFDL-driven timeslice/regional climate model.
  - Two-factor functional analysis of variance.

NARCCAP

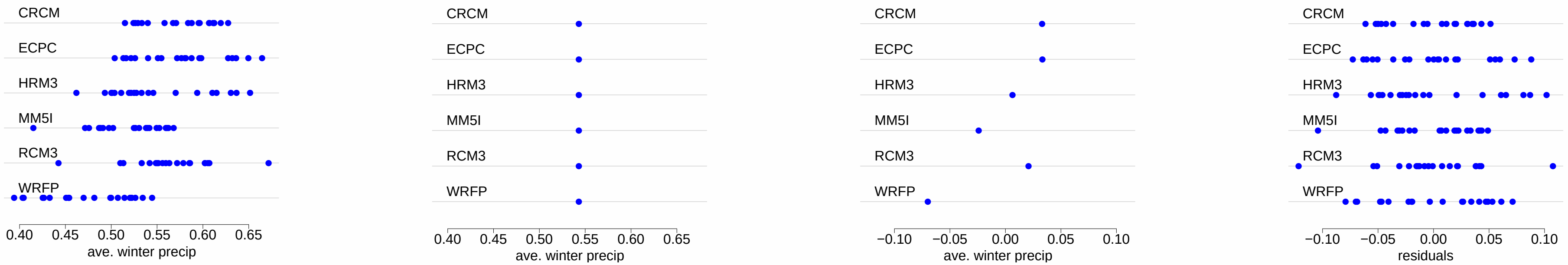
- North American Regional Climate Change Assessment Program (`www.narccap.ucar.edu`)
  - NCAR, ISU, CCCma, OURANOS, LLNL, GFDL, Hadley, Scripps, PNNL, USSC, etc.
  - NSF, NOAA, DOE, EPA
- *Systematically investigate the uncertainties in regional scale projections of future climate* and produce high resolution climate change projections using multiple RCM and multiple GCM simulations.
- 4 GCMs provide boundary conditions for 6 RCMs
  - balanced half-fraction

NCEP Experiment

- Six regional models
  - CRCM (OURANOS/UQAM), ECPC (UC San Diego/Scripps), HRM3 (Hadley Centre), MM5I (Iowa State U.), RCM3 (UC Santa Cruz), WRFP (PNNL)
- Boundary conditions supplied by NCEP Reanalysis II.
- 1981 – 2000 (20 years)
- Average daily precipitation (mm) – winter (DJF)
- Interpolated to a common grid: $120 \times 98 = 11,760$ grid boxes


CRCM
ECPC
HRM3
MM5I
RCM3
WRFP
Yr 1
Yr 2
Yr 3
⋮
Yr 20
30 40 50 60
-140 -100 -80 -60
0.0 0.2 0.4 0.6 0.8 1.0 1.2

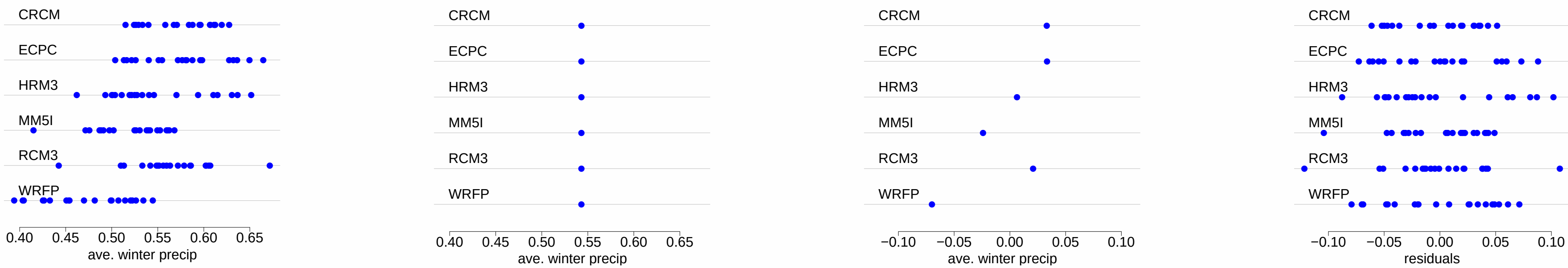
Analysis of Variance



$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

- For every grid box (this grid-box is in eastern Nebraska):
  - Y_{ij} is the response (transformed precipitation) for the ith model and the jth year.
  - μ is a common mean
  - α_i is a RCM-specific effect
  - ϵ_{ij} is the error or residual

Analysis of Variance



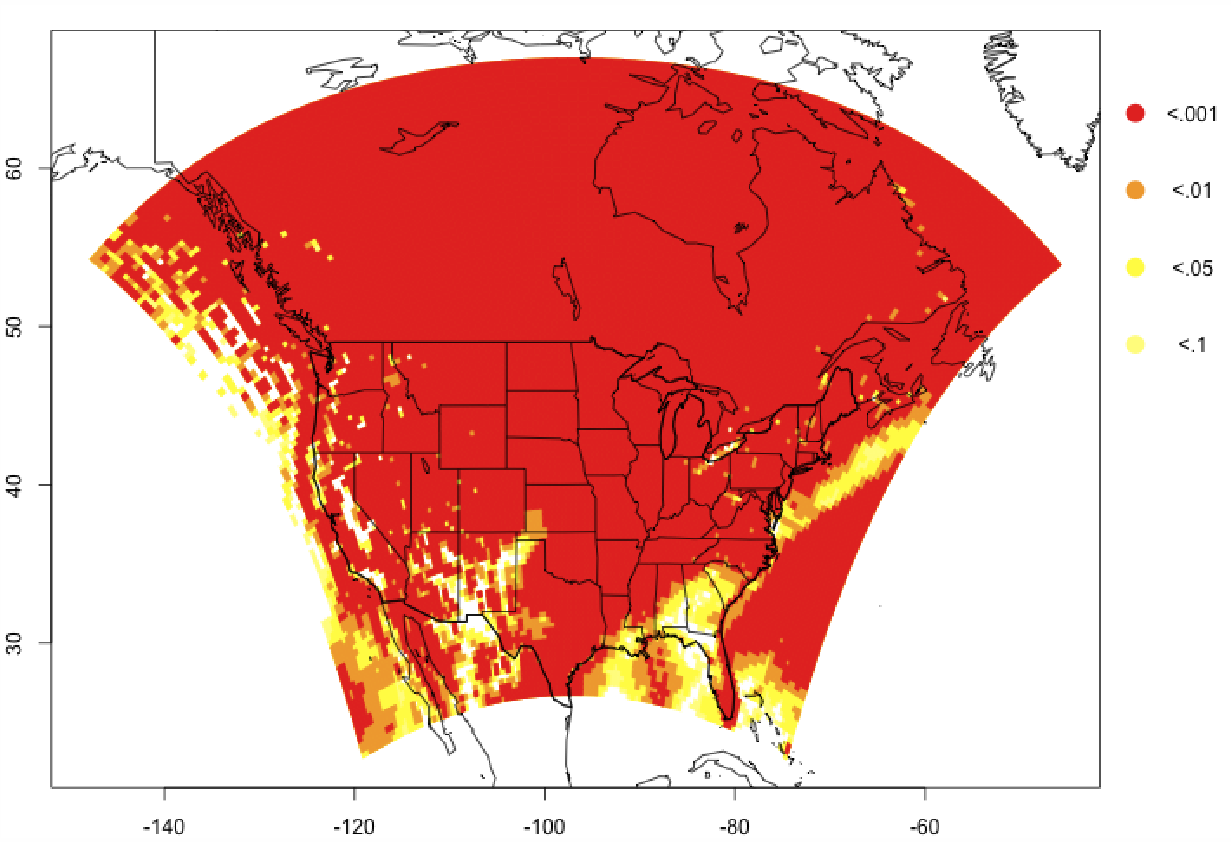
$$Y_{ij} \quad = \quad \mu \quad + \quad \alpha_i \quad + \quad \epsilon_{ij}$$

- Testing the null hypothesis $H_0 : \alpha_1 = \ldots = \alpha_6 = 0$:

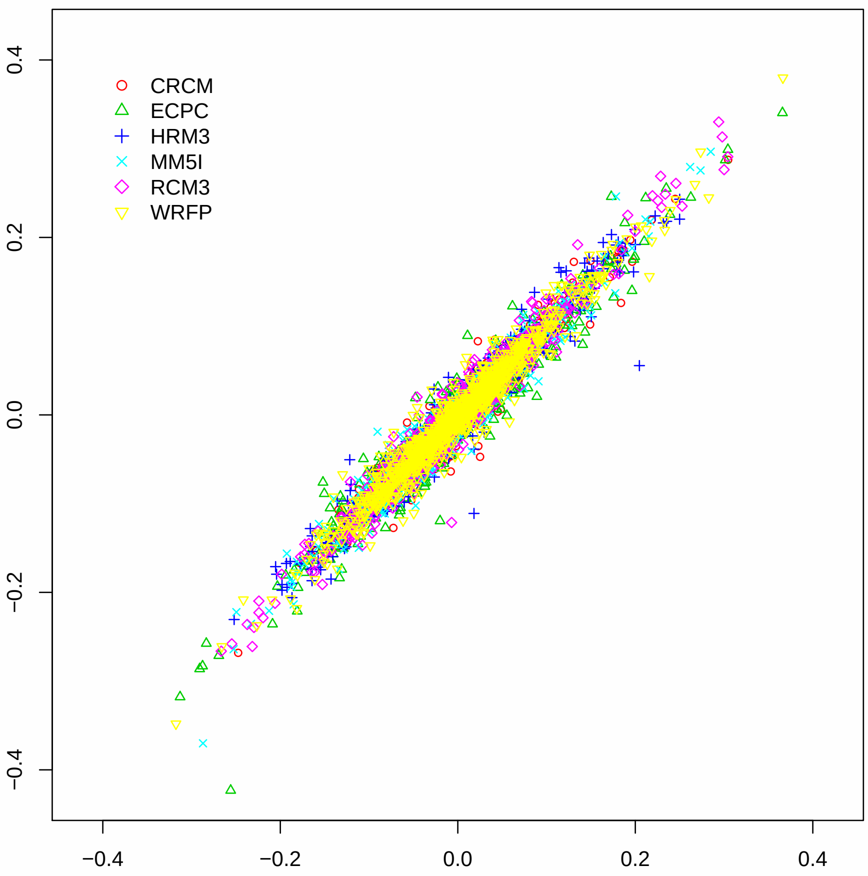
	df	SS	MS	F	p-value	
RCM	5	0.163	0.0326	15.3	1.75e-11	$\star\star\star$
Residual	114	0.243	0.00213			

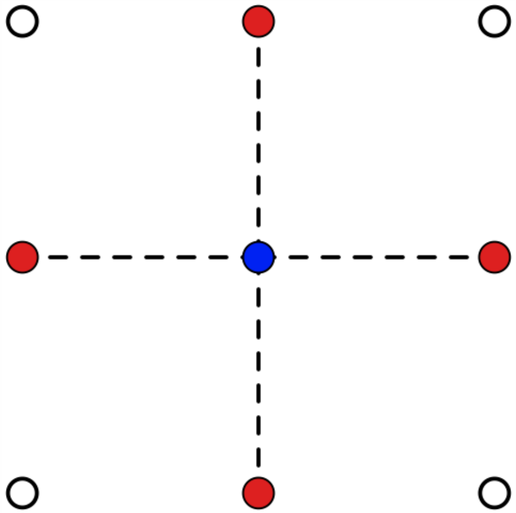
- Conclusion: strong evidence of differences in the RCM means.

Analysis of Variance



Map of pointwise p-values: strong evidence of differences in RCM means over nearly every grid box in the domain ???

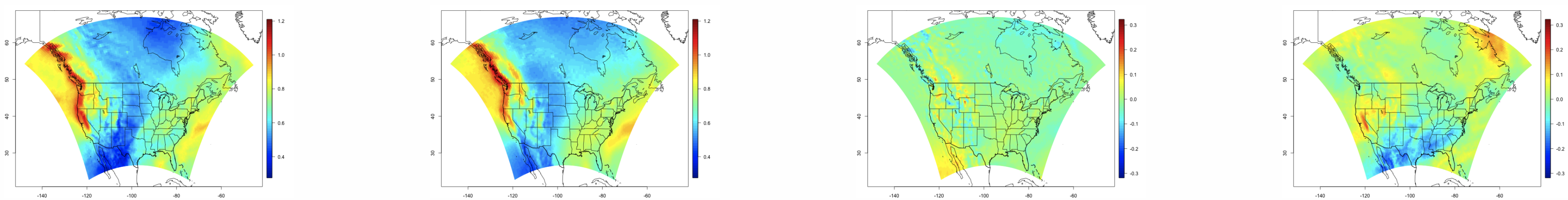




- Problem: correlated residuals at neighboring grid-boxes.

★ Result: invalid inference – any conclusions based on the p-value map are suspect.

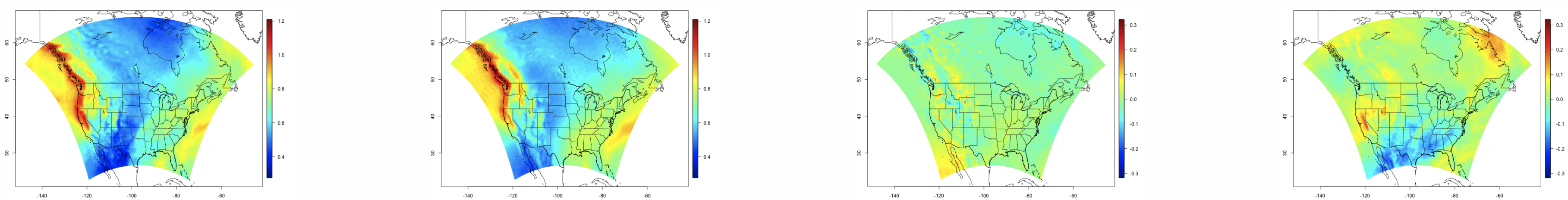
Functional Analysis of Variance



$$\mathbf{Y}_{ij} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\epsilon}_{ij}$$

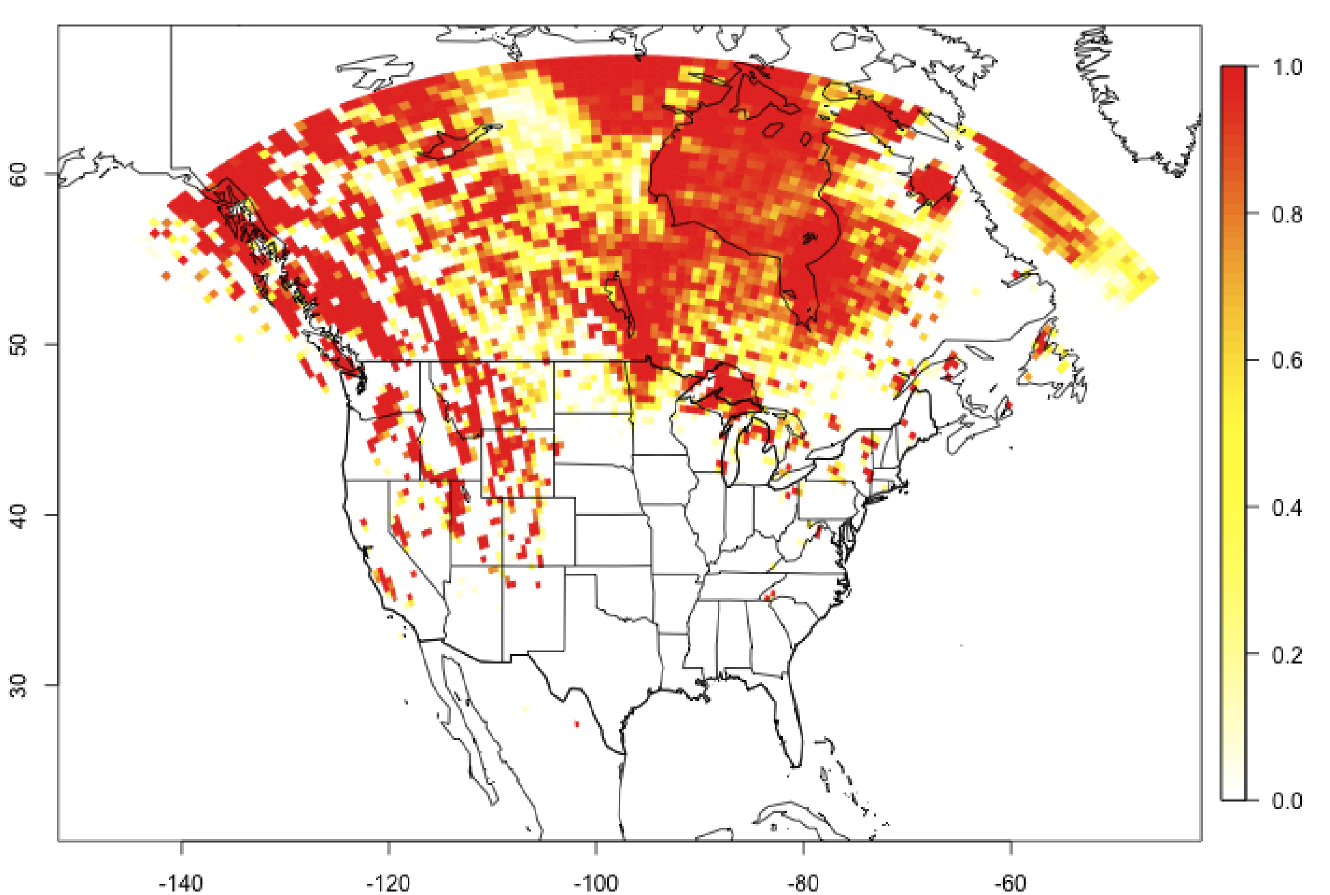
- The goal is to partition the variation into specific effects:
  - $\mathbf{Y}_{ij}$ is the vector response (transformed precipitation) for the ith model and jth year.
  - $\boldsymbol{\mu}$ is the vector mean common to all RCMs
  - $\boldsymbol{\alpha}_i$ is the vector RCM-specific effect
  - $\boldsymbol{\epsilon}_{ij}$ is the vector residual.

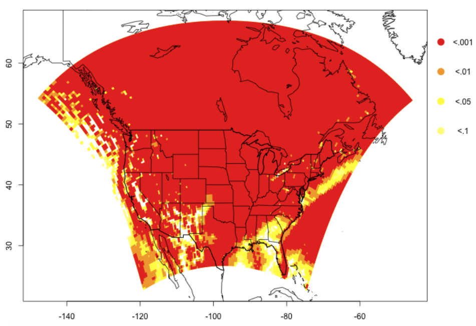
Functional Analysis of Variance



$$\mathbf{Y}_{ij} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\epsilon}_{ij}$$

- The innovation is that each of these effects is a *surface*.
- Each effect is considered a realization from a random process.
- Gaussian fields are often used as prior distributions; inferences about the effects involve conditioning on the observed output fields.
- Kaufman and Sain (2009, submitted).





$\widehat{P}[s_\alpha^2 > s_\epsilon^2]$

Pointwise probabilities that the model-to-model variation is larger than the year-to-year variation (analogous to small p-values in a traditional ANOVA).

A Statistical Model

- A common approach involves a three-level hierarchy:

Data model:	$[data \mid process, parameters]$
Process model:	$[process \mid parameters]$
Prior model:	$[parameters]$

- Simplifies the problem by factoring a complicated distribution into a series of conditional distributions.

- Inference involves sampling the posterior distribution:

$$[process, parameters \mid data] \propto [data \mid process, parameters][process \mid parameters][parameters]$$

A Statistical Model

- A hierarchical structure:

$$
\begin{aligned}
\text{Data model:} \quad & \mathbf{Y}_{ij} \sim \mathcal{N}\left(\boldsymbol{\mu}_i, \sigma_i^2 \mathbf{V}(\phi_i)\right), \quad i = 1, \ldots, 6, j = 1, \ldots, 20 \\
\text{Process model:} \quad & \boldsymbol{\mu}_i \sim \mathcal{N}\left(\boldsymbol{\mu}, \sigma^2 \mathbf{V}(\phi)\right) \\
\text{Prior model:} \quad & \boldsymbol{\mu} \sim \mathcal{N}\left(\textbf{NCEP}, \sigma_{\boldsymbol{\mu}}^2 \mathbf{V}(\phi_{\boldsymbol{\mu}})\right)
\end{aligned}
$$

  - $\{\mathbf{Y}_{ij}\}$ are (transformed) daily average precipitation fields
  - $\{\boldsymbol{\mu}_i\}$ are model specific means; $\boldsymbol{\mu}$ is the “grand” mean
  - $\{\sigma_i^2\}$, σ^2, $\sigma_{\boldsymbol{\mu}}^2$ are scale parameters
  - $\{\phi_i\}$, ϕ, $\phi_{\boldsymbol{\mu}}$ are spatial dependence parameters

- Prior distributions on scale and spatial dependence parameters are non-informative.

A Statistical Model

- An alternative (ANOVA) formulation:

$$\mathbf{Y}_{ij} = \textbf{NCEP} + \boldsymbol{\eta} + \boldsymbol{\alpha}_i + \boldsymbol{\epsilon}_{ij}, \quad i = 1, \ldots, 6, j = 1, \ldots, 20$$

  - $\boldsymbol{\eta}$ is a common component to all fields and explains variation beyond NCEP.
    - $\boldsymbol{\mu} = \textbf{NCEP} + \boldsymbol{\eta}$.
  - $\{\boldsymbol{\alpha}_i\}$ are RCM-specific components.
    - $\boldsymbol{\mu}_i = \textbf{NCEP} + \boldsymbol{\eta} + \boldsymbol{\alpha}_i$.
  - $\{\boldsymbol{\epsilon}_{ij}\}$ represent year-to-year variation.

- Sain, Kaufman, and Tebaldi (2009, in preparation)

NCEP Experiment

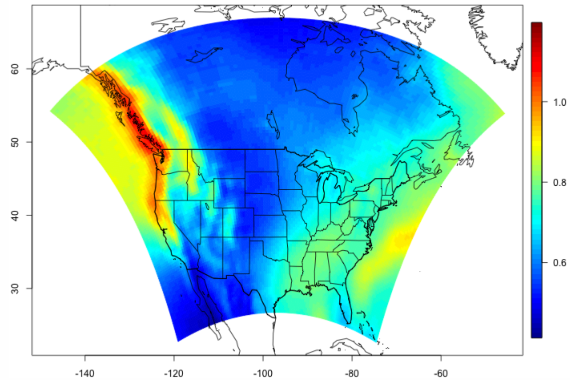
- Recall the functional ANOVA model:

$$\mathbf{Y}_{ij} = \textbf{NCEP} + \boldsymbol{\eta} + \boldsymbol{\alpha}_i + \boldsymbol{\epsilon}_{ij}, \;\; i = 1, \ldots, 6, j = 1, \ldots, 20$$

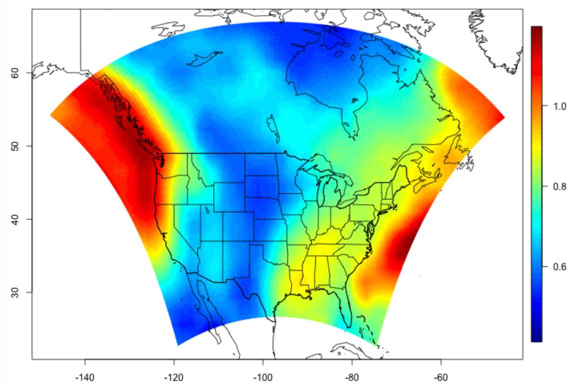
- Compare $\boldsymbol{\mu}$ to NCEP – how do the RCMs on average compare to the driving model?

- Compare $\{\boldsymbol{\alpha}_i\}$ – how consistent are the RCMs and how do they compare with each other?

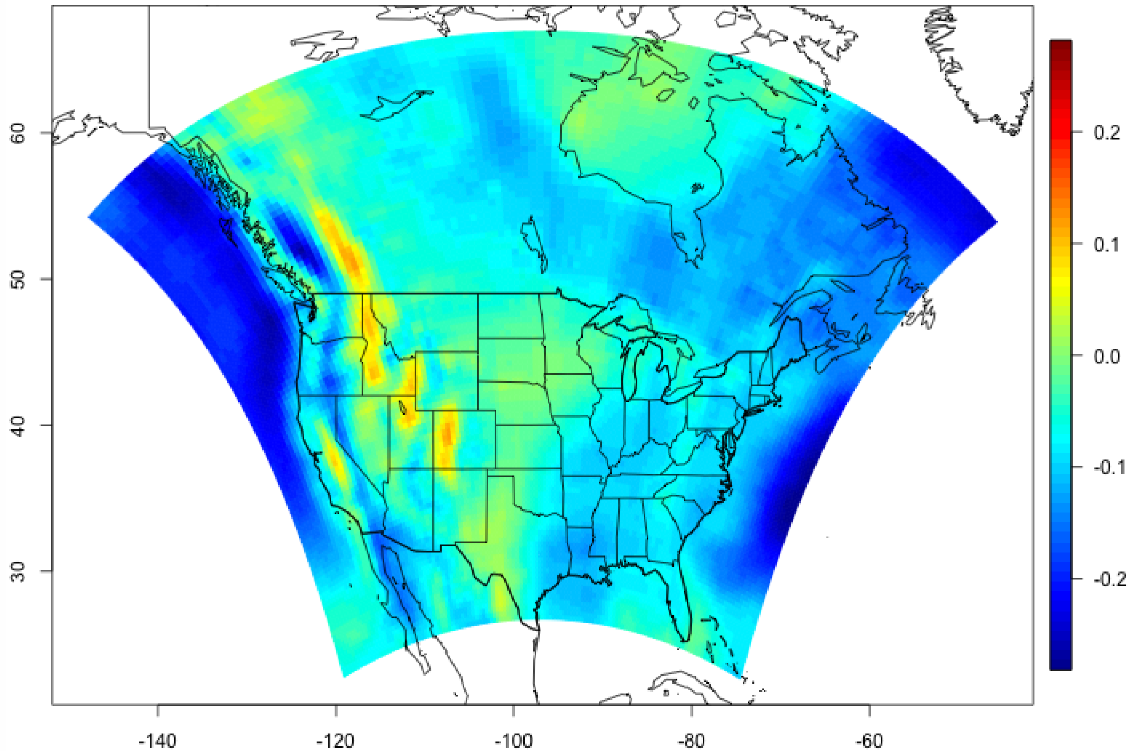
- By drawing samples from the posterior (ensemble), we can address these questions giving insight to the sources of uncertainty in the collection of RCM output.



$\boldsymbol{\mu}$

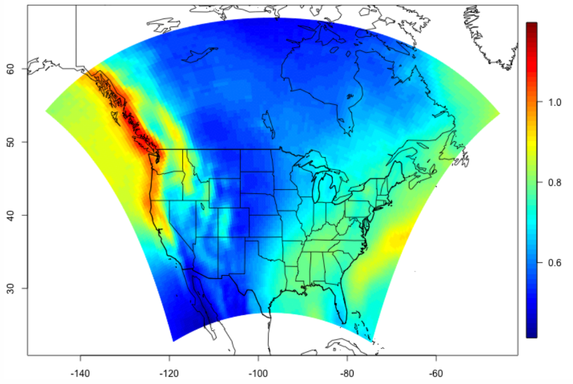


NCEP

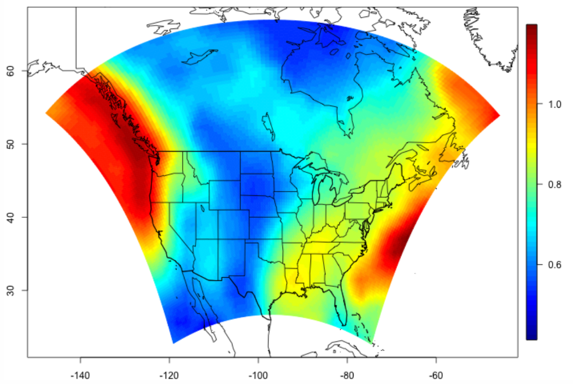


$\boldsymbol{\mu}$ − NCEP

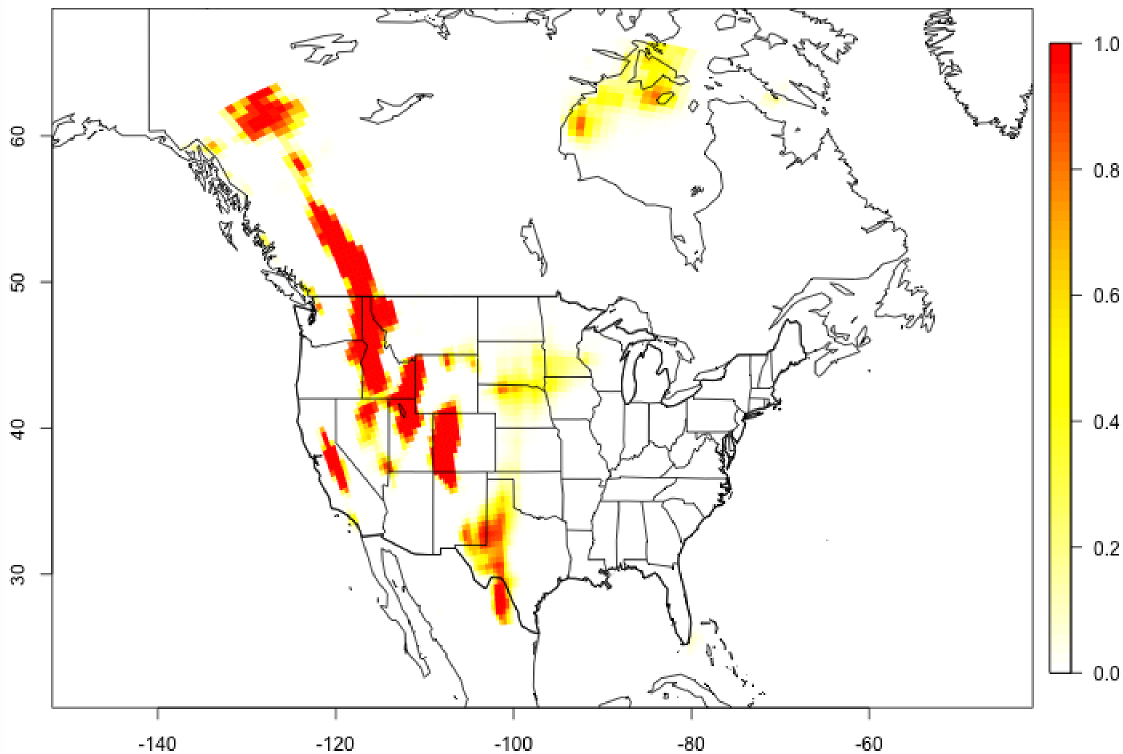
- Difference between posterior mean for $\boldsymbol{\mu}$ and the mean NCEP.
- Average daily winter precipitation (transformed).



$\boldsymbol{\mu}$

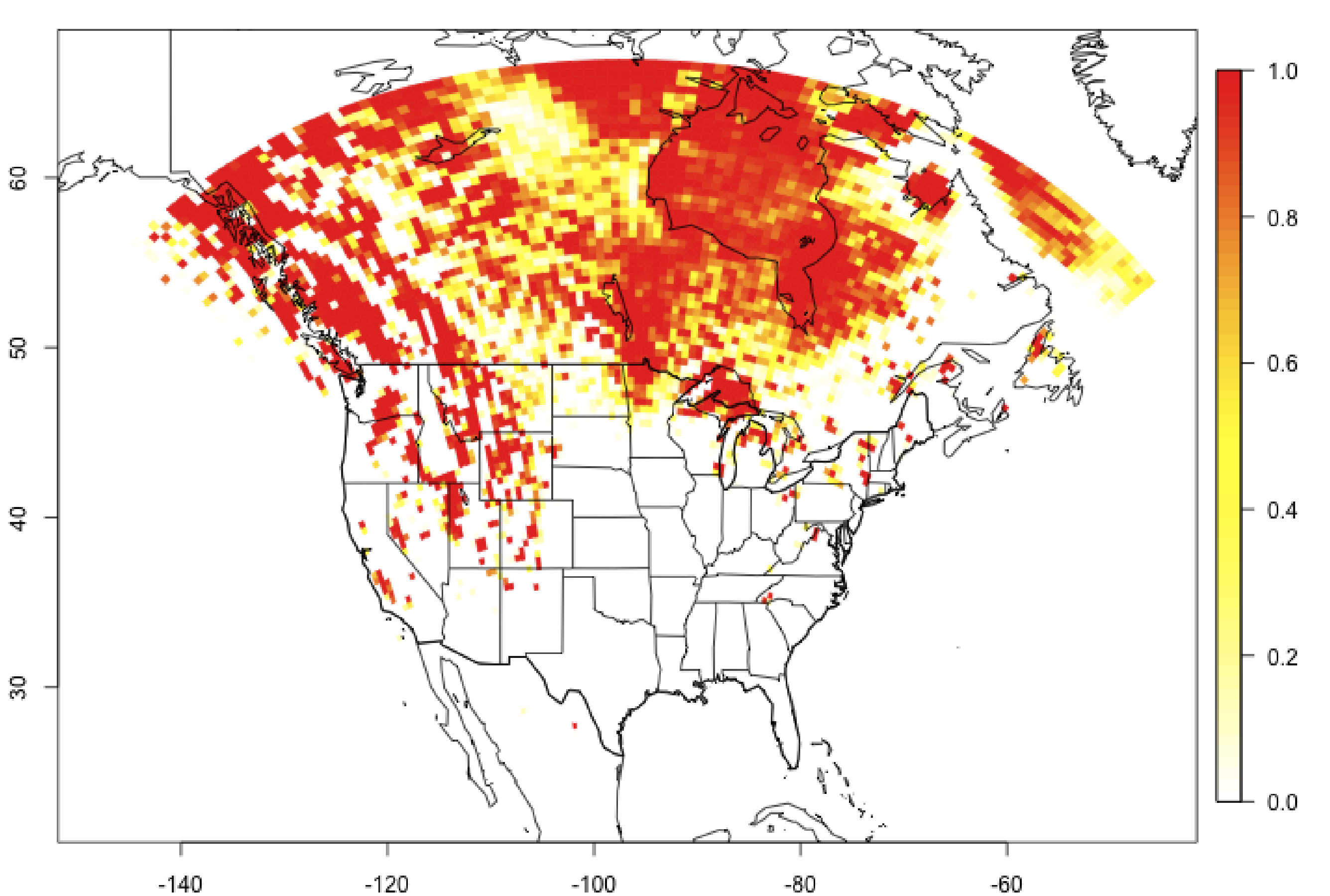


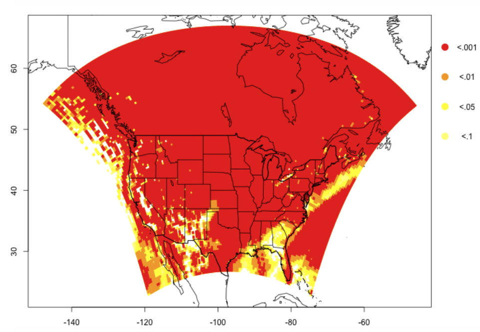
NCEP



$\widehat{P}[\mu > \text{NCEP}]$

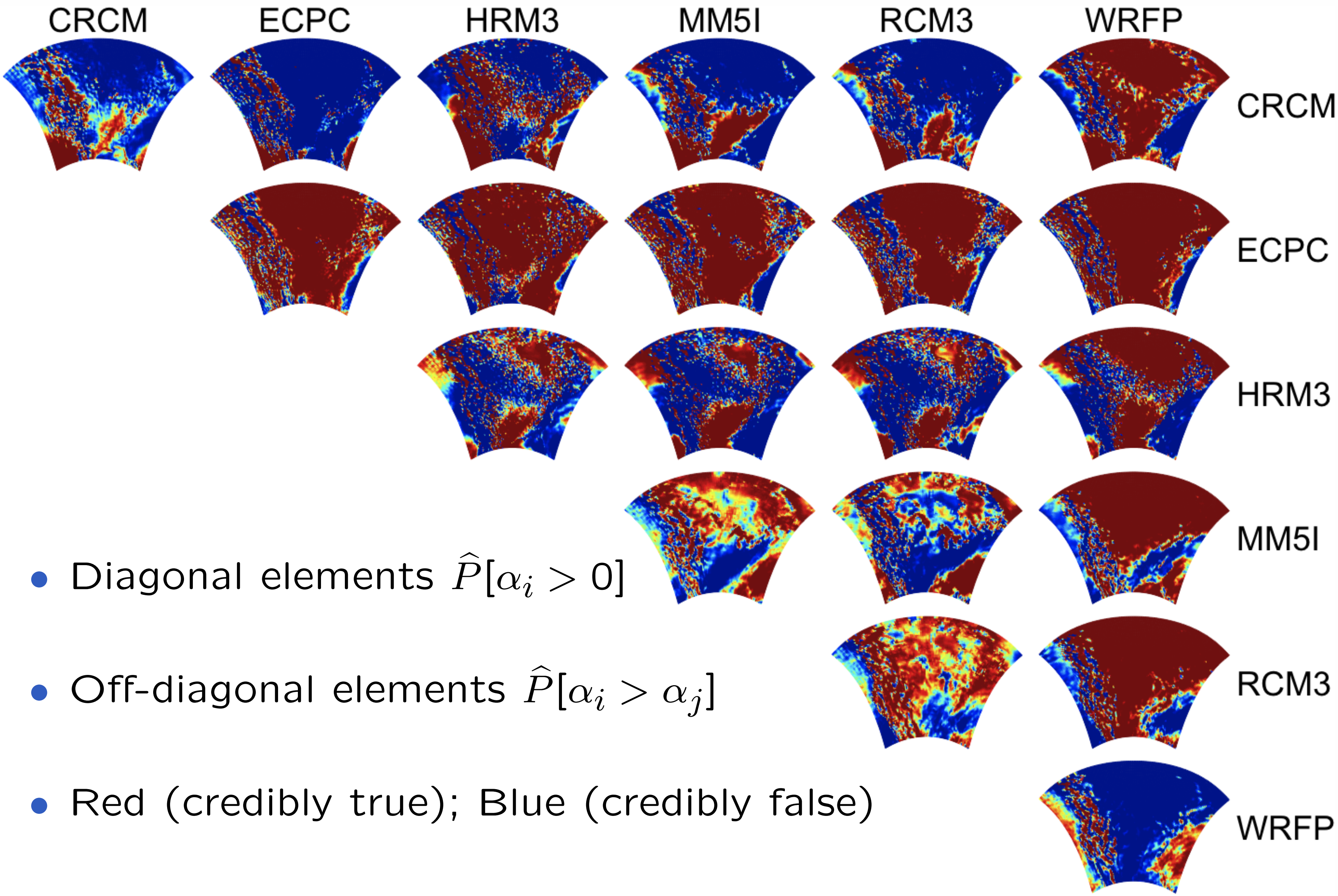
Pointwise probabilities that draws from the posterior distribution of $\boldsymbol{\mu}$ are greater than the mean NCEP field. Red (credibly true); white (credibly false).





$$\widehat{P}[s_\alpha^2 > s_\epsilon^2]$$

Pointwise probabilities that the model-to-model variation is larger than the year-to-year variation.



- Diagonal elements $\widehat{P}[\alpha_i > 0]$
- Off-diagonal elements $\widehat{P}[\alpha_i > \alpha_j]$
- Red (credibly true); Blue (credibly false)

Heat Stress: A Preliminary Study

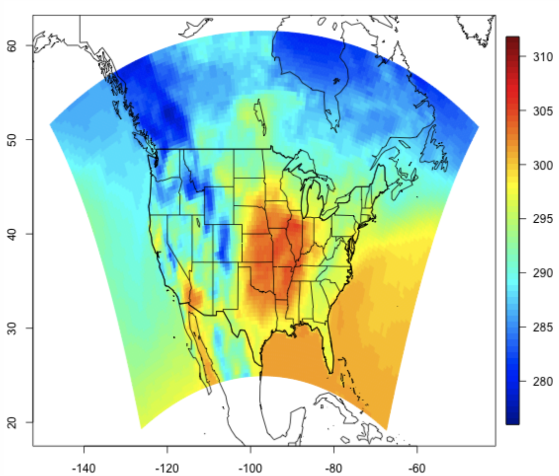
- Two types of dynamic downscaling: a GFDL time-slice and a GFDL-driven RCM (RCM3; UC Santa Cruz).
  - Geophysical Fluid Dynamics Laboratory (GFDL; NOAA)
- Both timeslice and RCM use the A2 scenario.
- Current (1971-2000) and future (2041-2070) runs.
- Focus on summer (May-September) heat stress.
  - Output interpolated to a common grid (134×83).
- Examine differences in the two models as well as changing heat stress in North America.

What is a Heat Wave/Heat Stress?

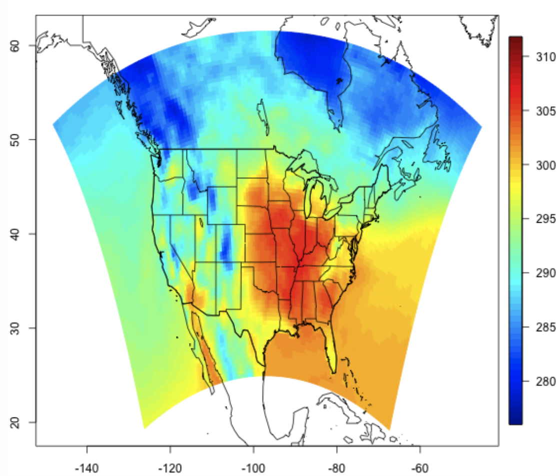
- "...an extended period of unusually high atmosphere-related heat stress, which causes temporary modifications in lifestyle, and which may have adverse health consequences for the affected population."
  - Intensity and duration and local climatology.

- We adopt the definition of heat stress put forth by Meehl and Tebaldi (2004) in their study of global climate models: maximum of the 3-day running mean of the overnight minimum temperature.
  - Captures persistence and (lack of) overnight cooling.

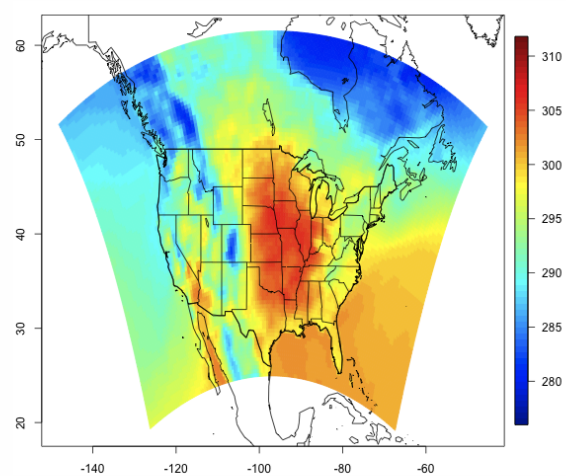
Heat Stress (Timeslice)



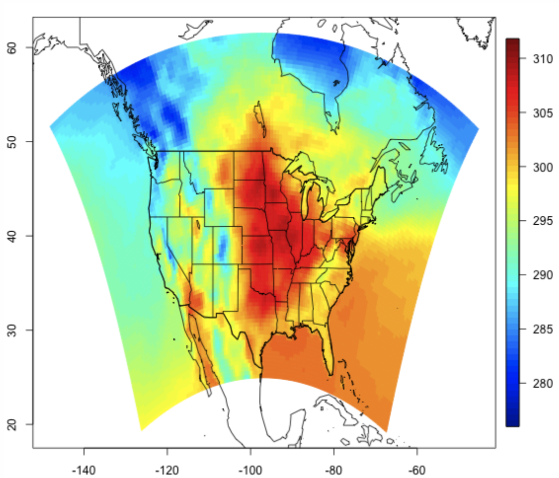
1980



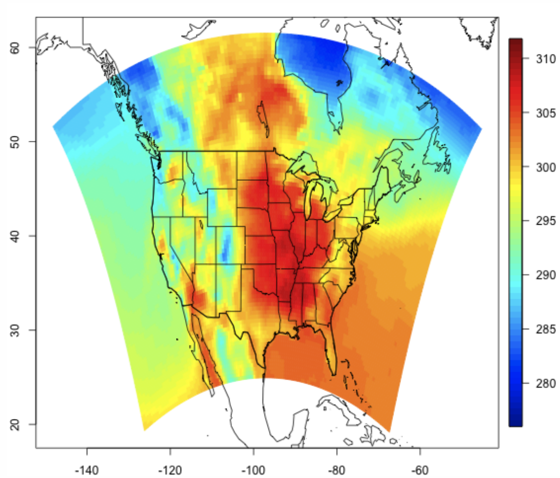
1990



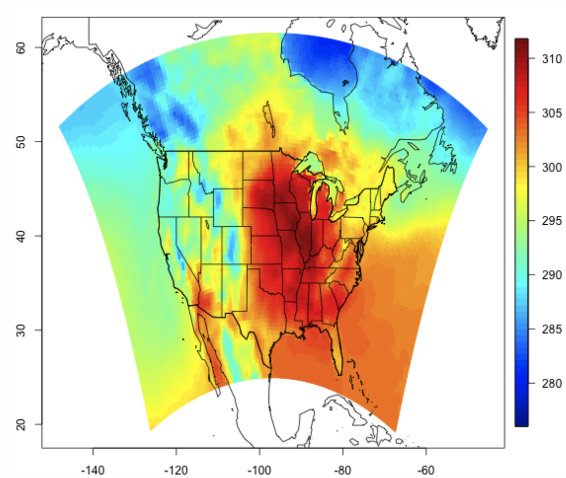
2000



2050

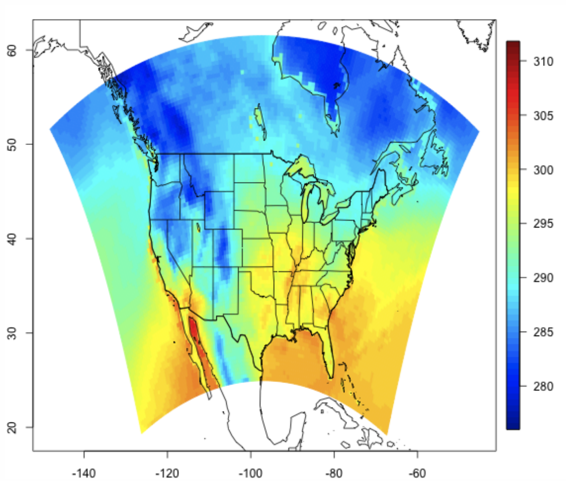


2060

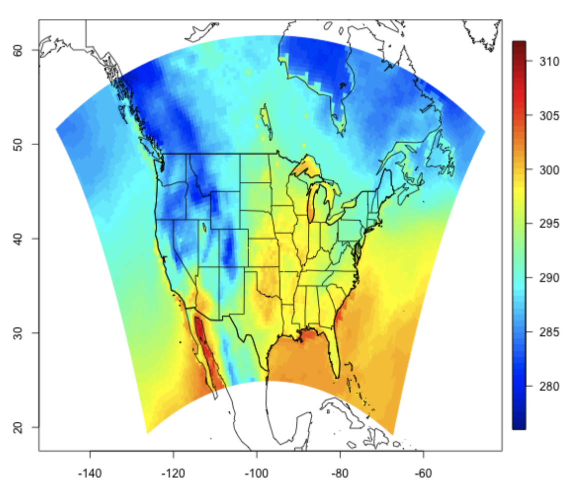


2070

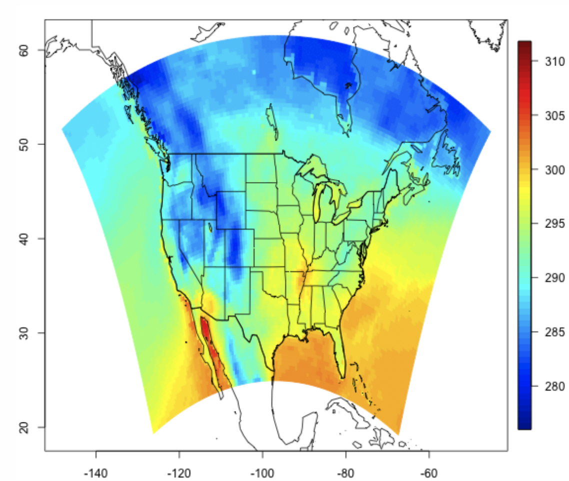
Heat Stress (RCM)



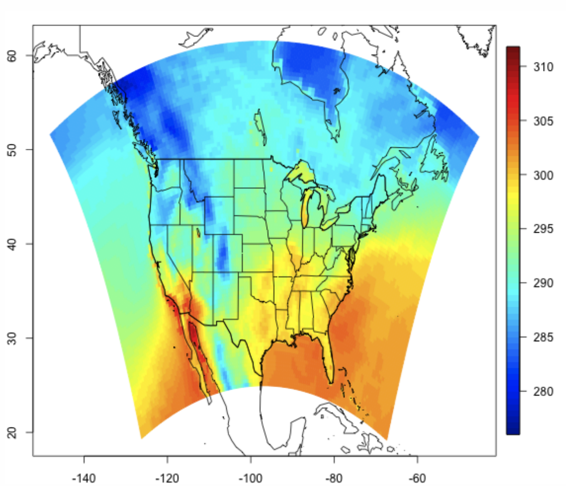
1980



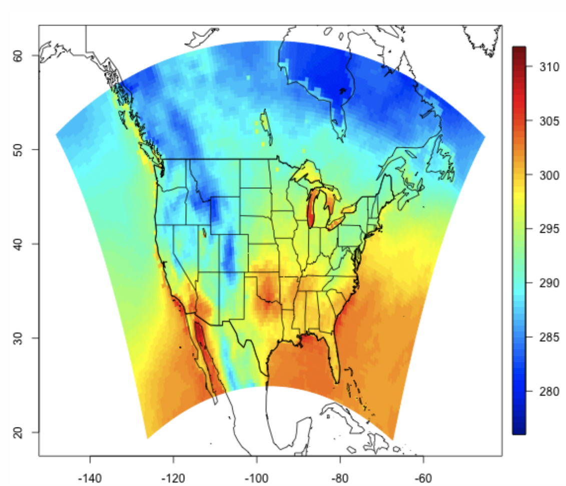
1990



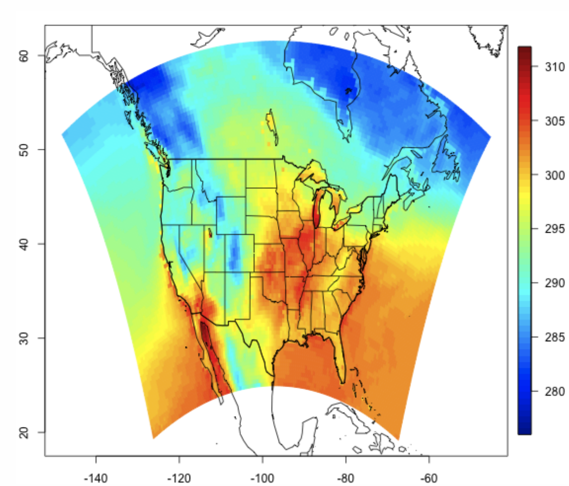
2000



2050

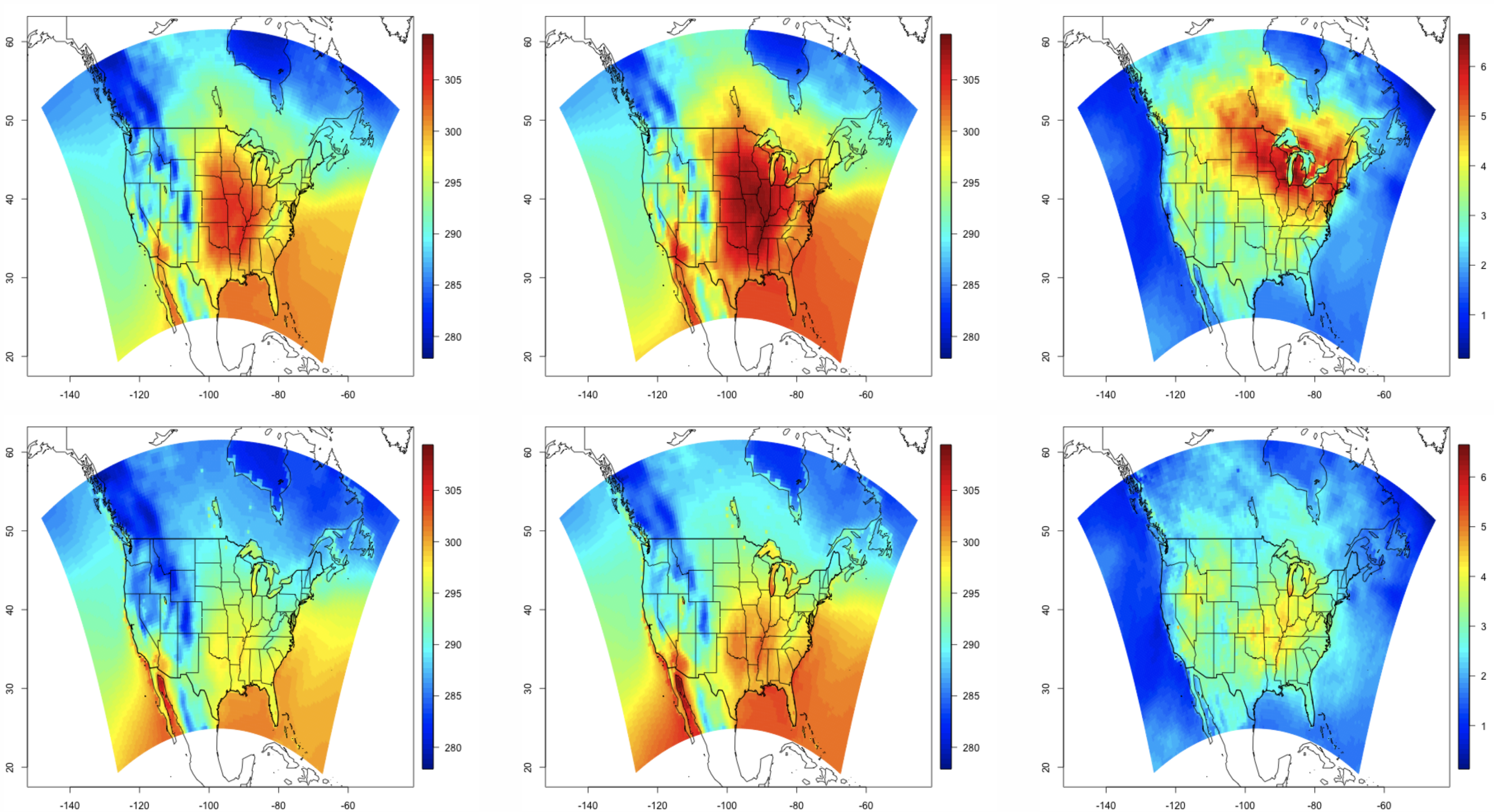


2060



2070

Heat Stress (Means)



Current Future Difference

A Functional ANOVA Model

- Let $\mathbf{Y}_{ijt}$ denote the ith model (timeslice vs RCM; $i = 0, 1$), the jth run (current vs future; $j = 0, 1$), at the tth time ($t = 1, \ldots 30$):

$$\mathbf{Y}_{ijt} = \boldsymbol{\alpha}_0 + i\boldsymbol{\alpha}_1 + j\boldsymbol{\alpha}_2 + ij\boldsymbol{\alpha}_3 + \boldsymbol{\epsilon}_{ijt}$$

- Assume each component is generated from a Markov random field:

$$\boldsymbol{\alpha}_0 \sim \mathcal{N}\left(\boldsymbol{\mu}_{curr}, \boldsymbol{\Sigma}(\boldsymbol{\theta}_0)\right) \qquad \boldsymbol{\alpha}_1 \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}(\boldsymbol{\theta}_1)\right)$$
$$\boldsymbol{\alpha}_2 \sim \mathcal{N}\left(\boldsymbol{\mu}_{diff}, \boldsymbol{\Sigma}(\boldsymbol{\theta}_2)\right) \qquad \boldsymbol{\alpha}_3 \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}(\boldsymbol{\theta}_3)\right)$$

- $\boldsymbol{\mu}_{curr}$ and $\boldsymbol{\mu}_{diff}$ are average fields of the current and difference in heat stress computed from the driving global GFDL model.

A Functional ANOVA Model

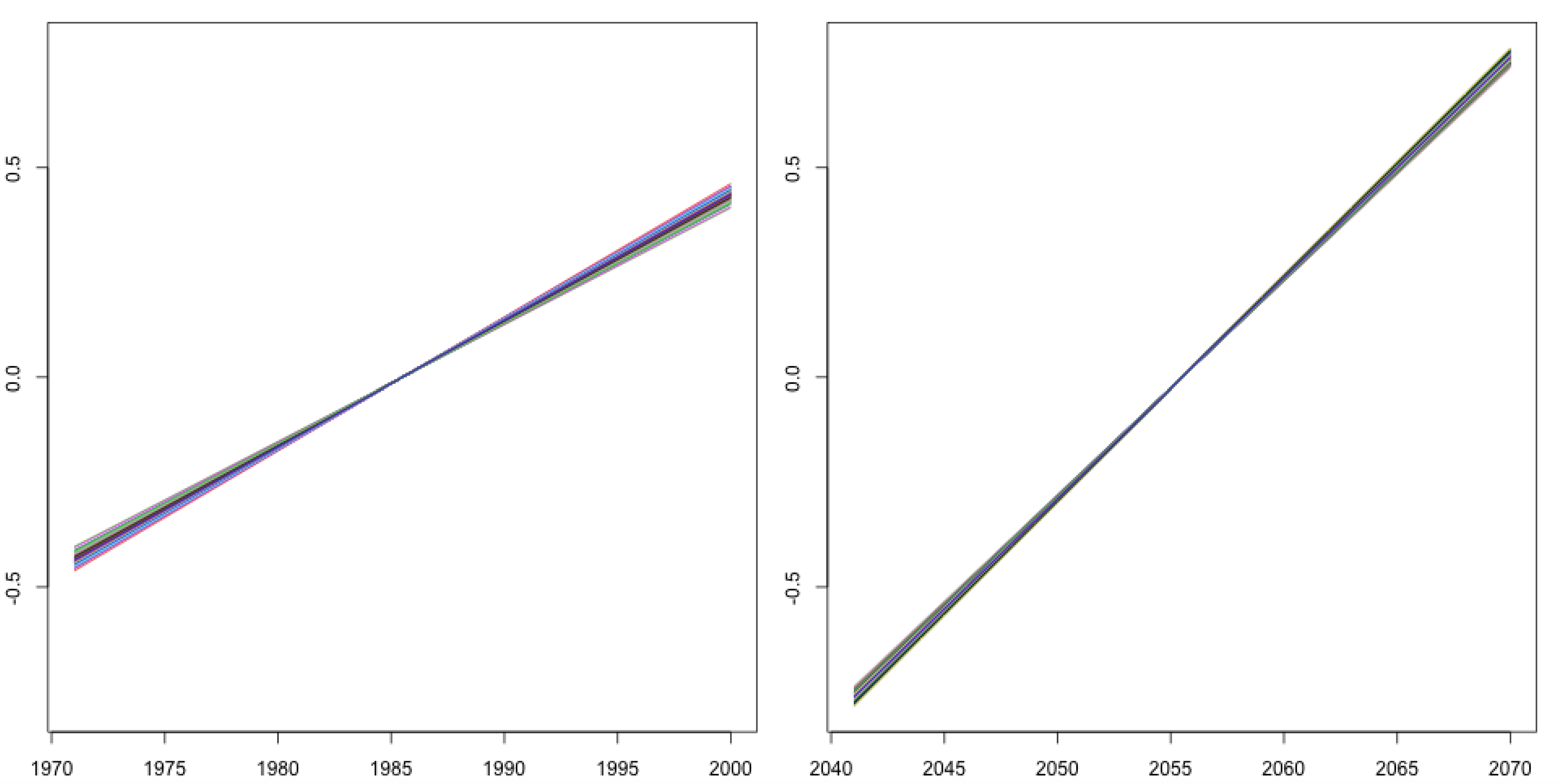
- The error term, $\boldsymbol{\epsilon}_{ijt}$ is broken up into two pieces:

$$\boldsymbol{\epsilon}_{ijt} = \gamma_j(t - 15.5) + \boldsymbol{\eta}_t$$

where

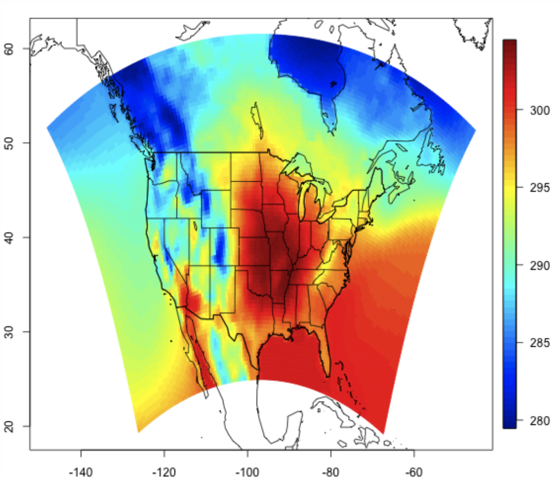
$$\gamma_j \sim \mathcal{N}\left(\gamma_j^*, \sigma_\gamma^2\right) \qquad \boldsymbol{\eta}_t \sim \mathcal{N}\left(0, \boldsymbol{\Sigma}(\boldsymbol{\theta}_t)\right)$$

- γ_j^* are average slopes from the control and future runs of the driving global GFDL model.

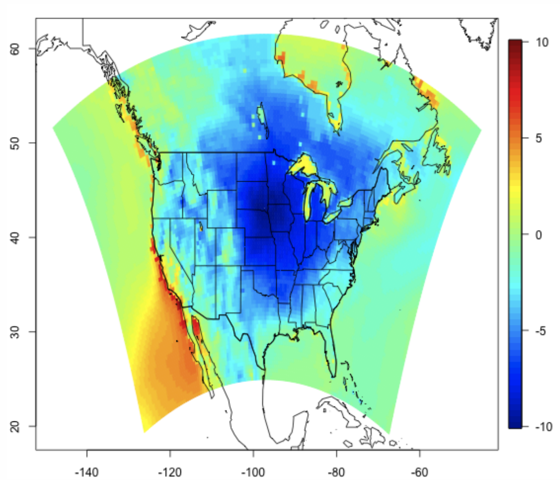


100 draws from the posterior of γ_0 (left) and γ_1 (right).

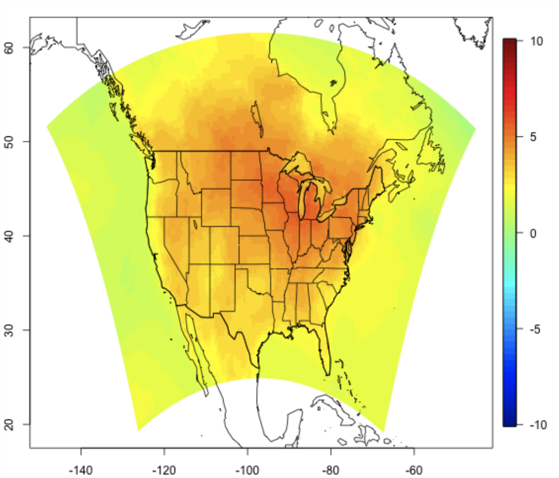
Posterior Means



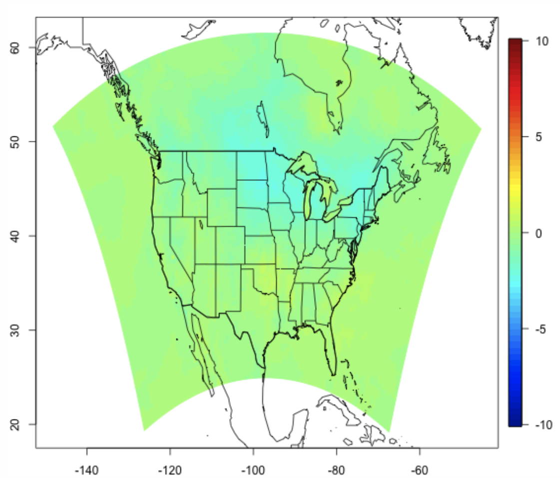
$\bar{\alpha}_0$



$\bar{\alpha}_1$



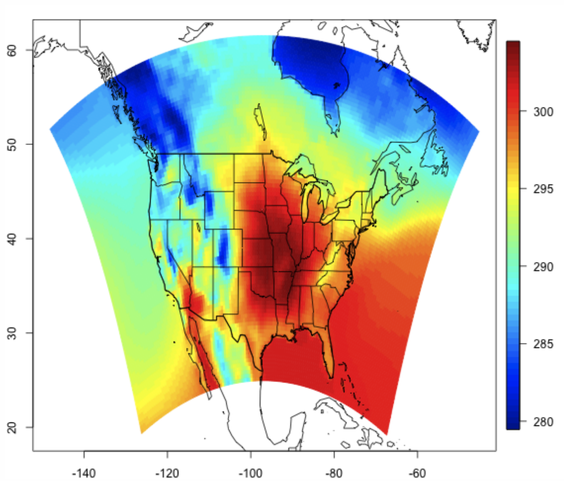
$\bar{\alpha}_2$



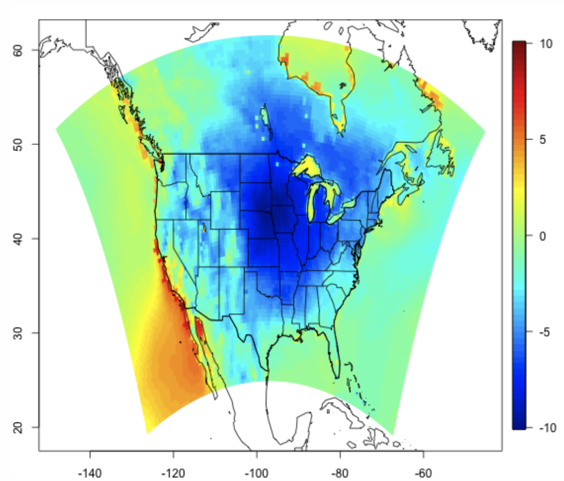
$\bar{\alpha}_3$

- α_0 represents current timeslice.
- α_1 adjusts for current RCM.
- α_2 adjusts for future run.
- α_3 is an interaction.

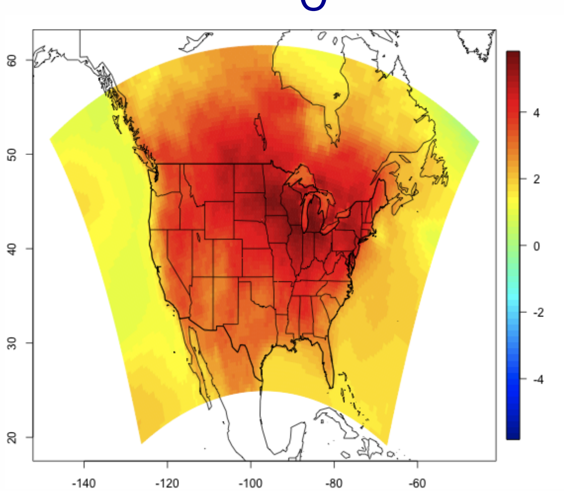
Posterior Means



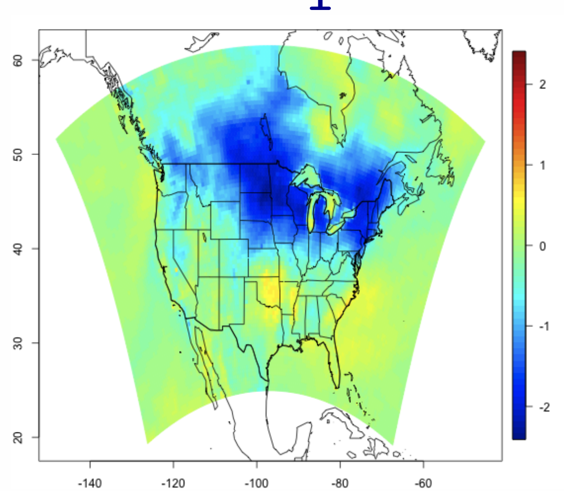
$\bar{\alpha}_0$



$\bar{\alpha}_1$



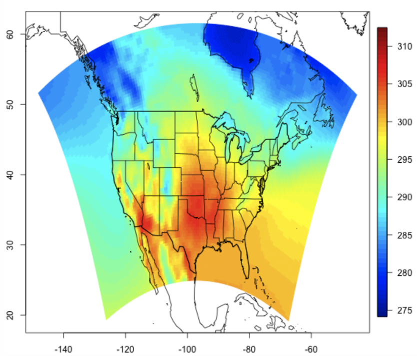
$\bar{\alpha}_2$



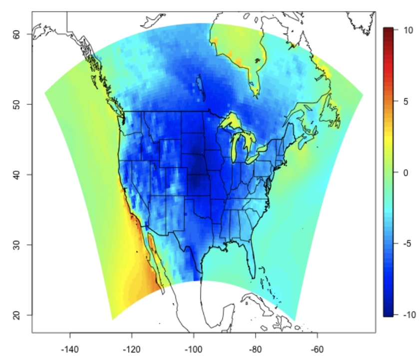
$\bar{\alpha}_3$

- α_0 represents current timeslice.
- α_1 adjusts for current RCM.
- α_2 adjusts for future run.
- α_3 is an interaction.

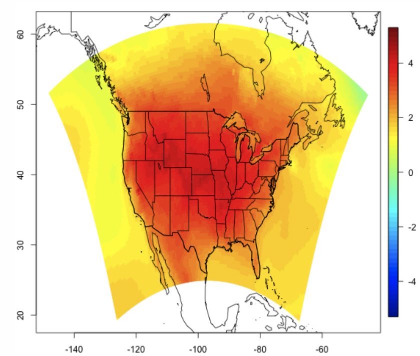
A Quick Look at Temperatures



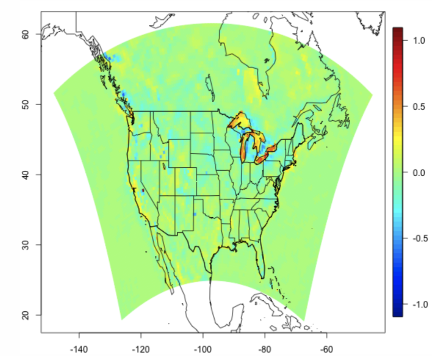
$\bar{\boldsymbol{\alpha}}_0$



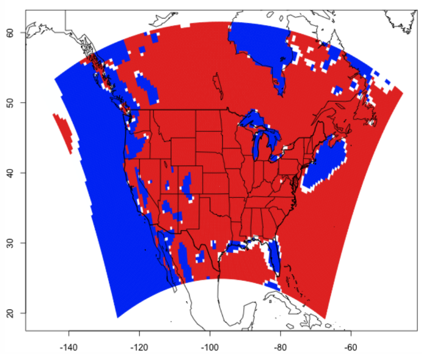
$\bar{\boldsymbol{\alpha}}_1$



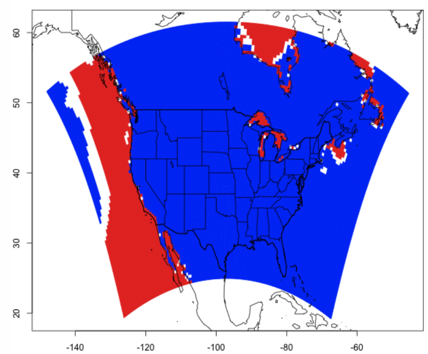
$\bar{\boldsymbol{\alpha}}_2$



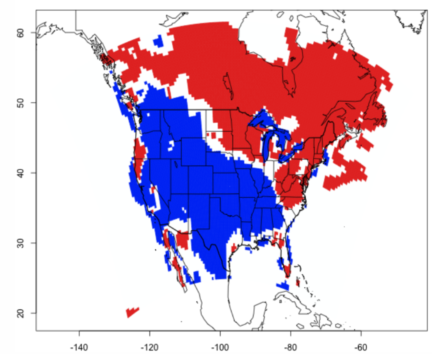
$\bar{\boldsymbol{\alpha}}_3$



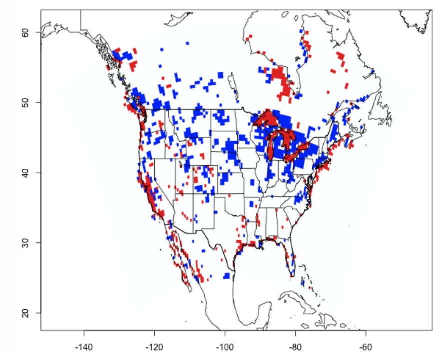
$\widehat{P}[\alpha_0 > \mu_{curr}]$



$\widehat{P}[\alpha_1 > 0]$

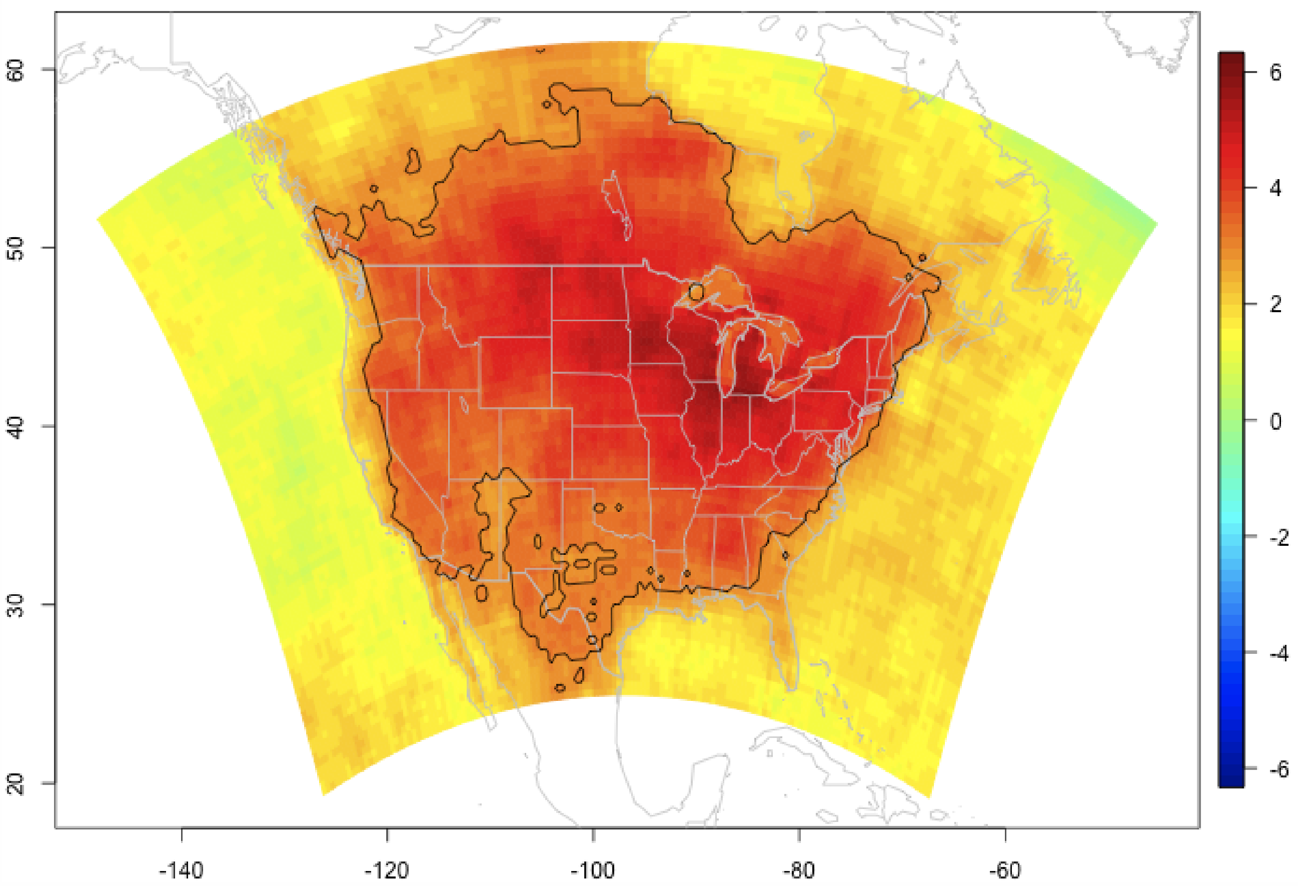


$\widehat{P}[\alpha_2 > \mu_{diff}]$

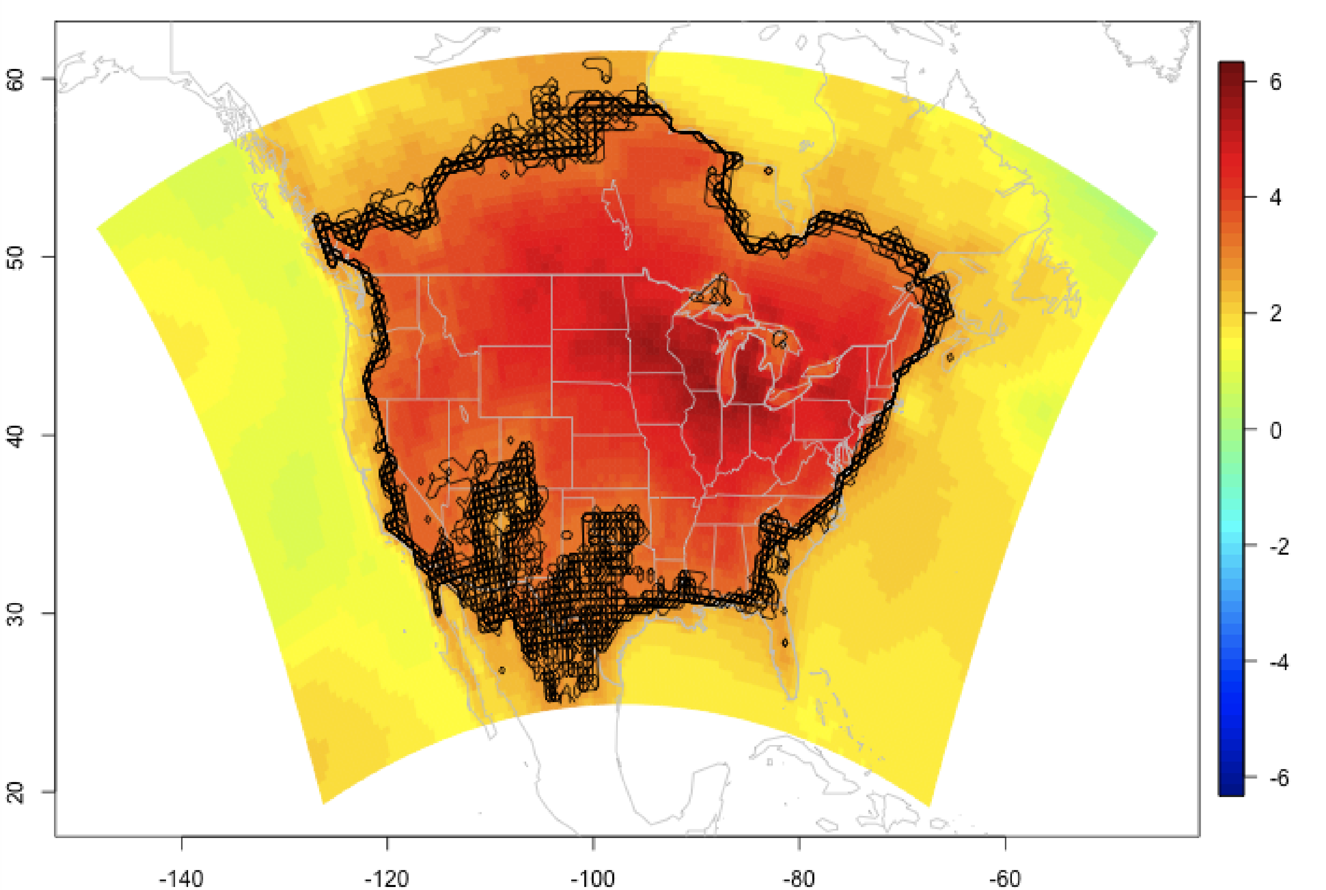


$\widehat{P}[\alpha_3 > 0]$

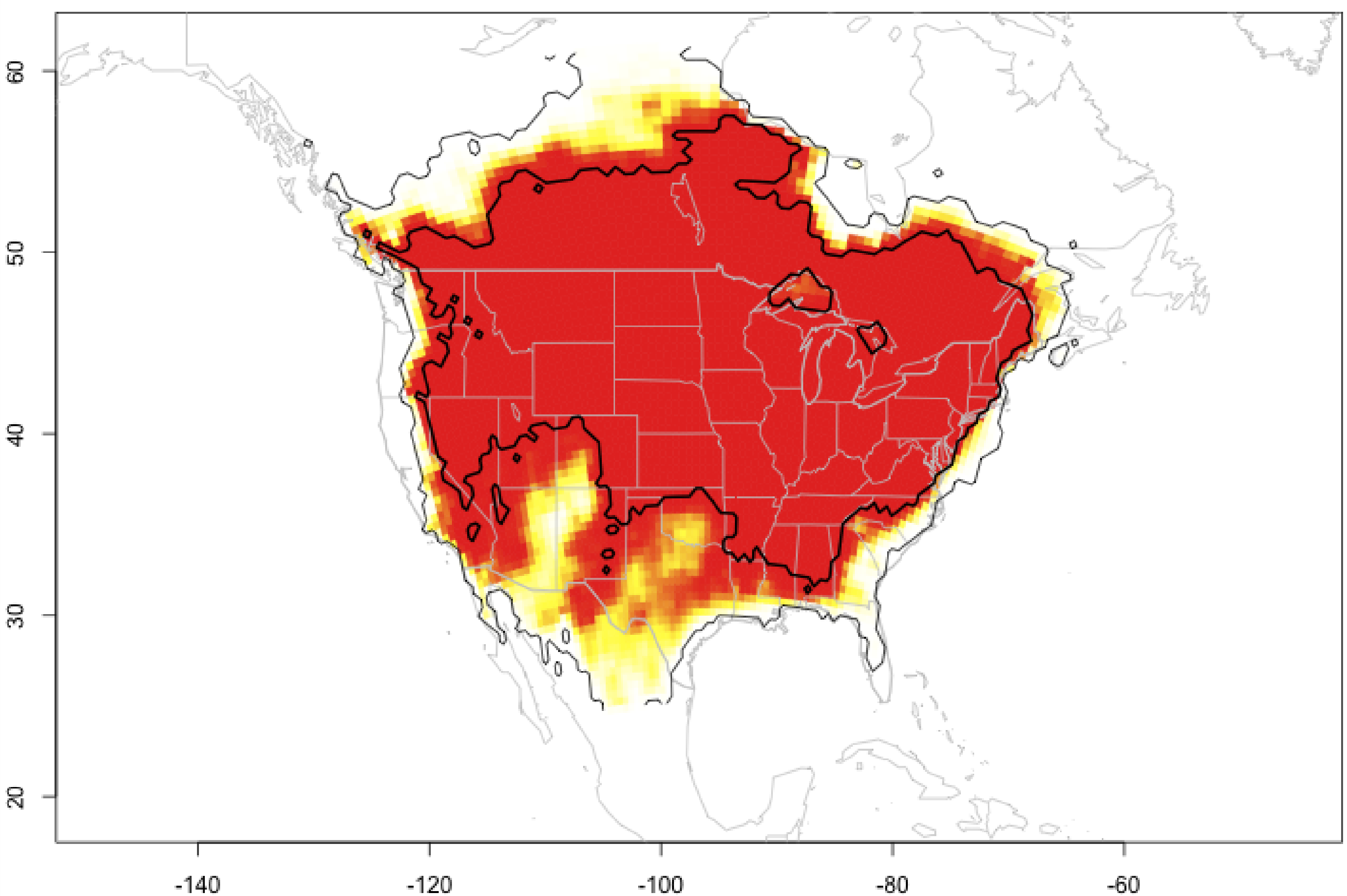
JJA Ave Temp – $p < 0.05$ – blue; $p > 0.95$ – red.



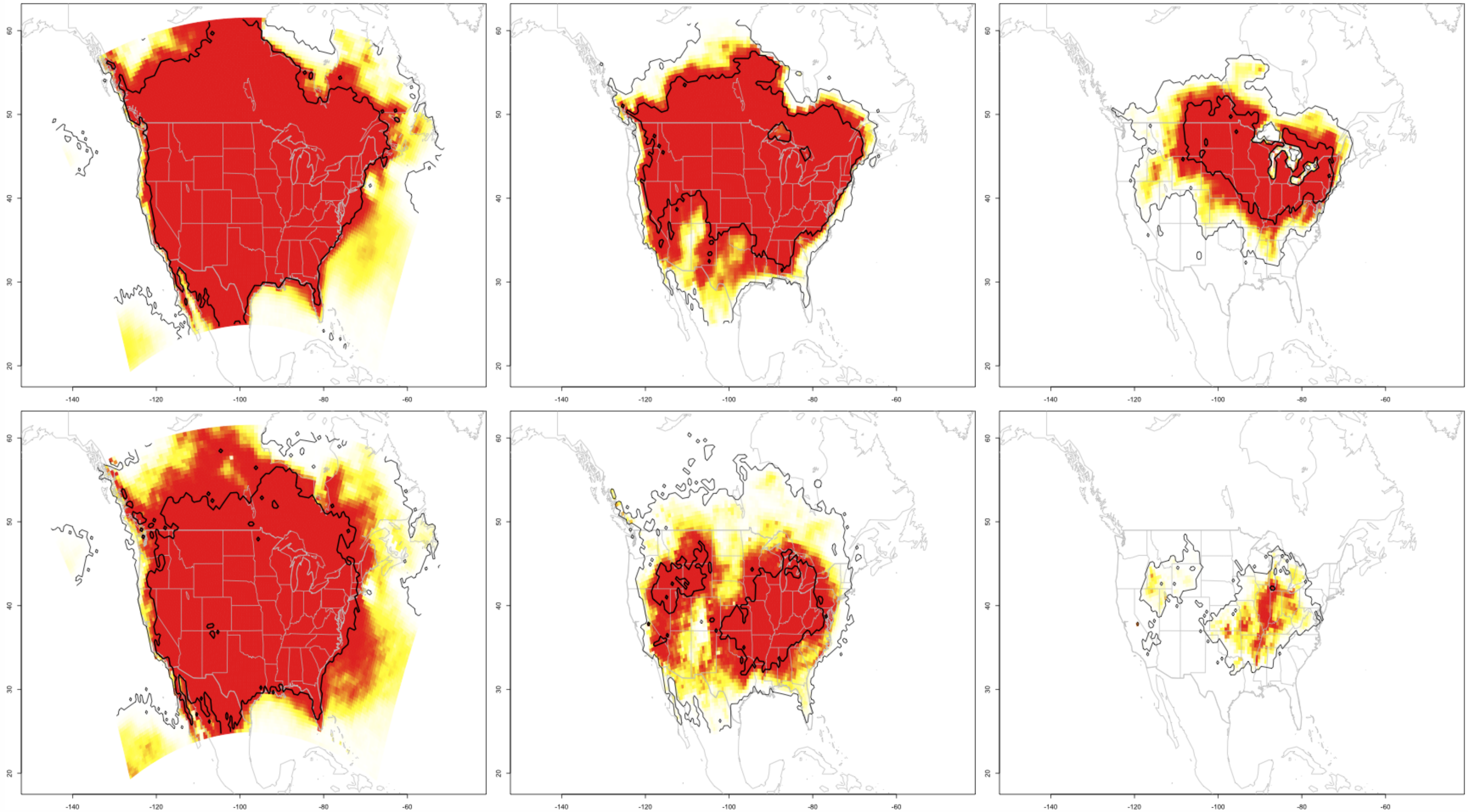
- A single draw from the posterior for α_2.
- Contour represents an increase in heat stress by 3.0 degrees.



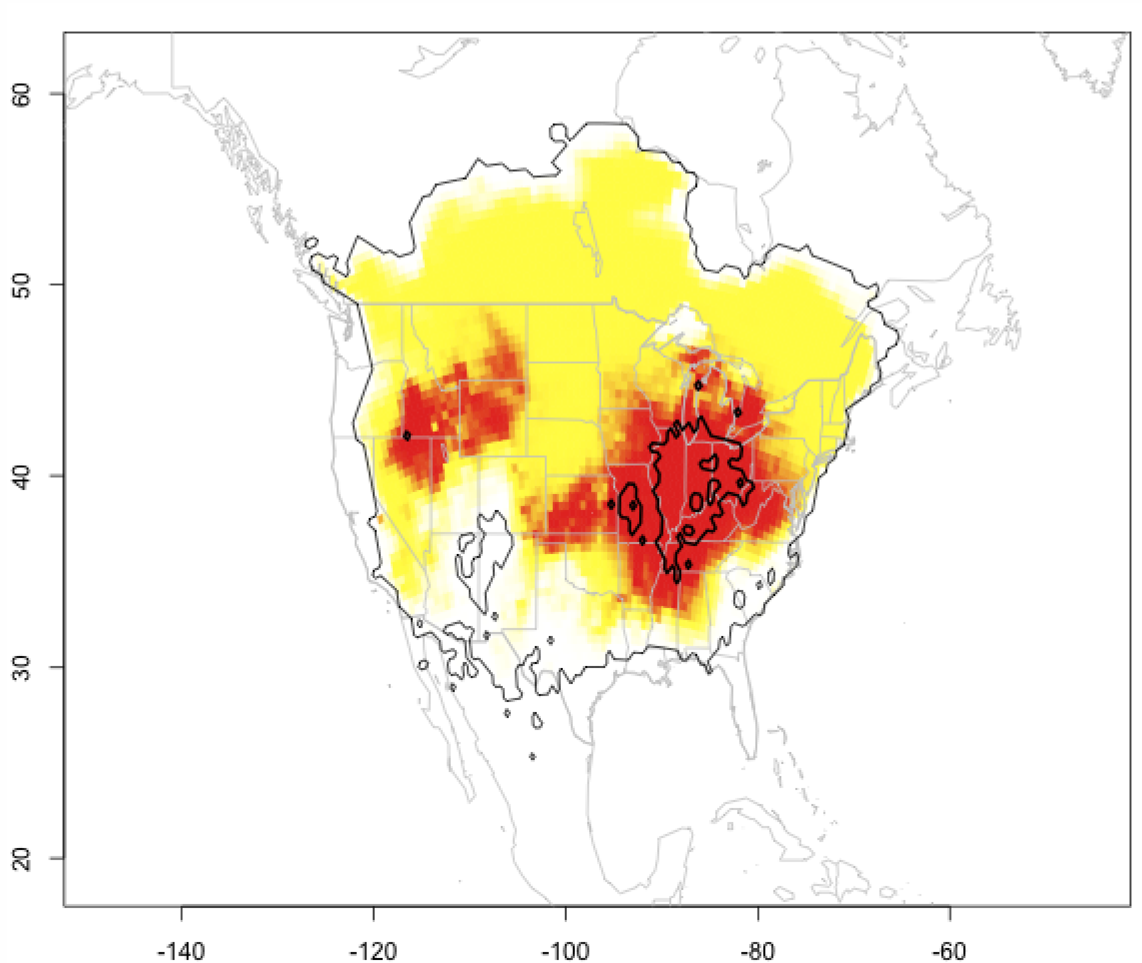
- Posterior mean for α_2.
- Contour lines represent an increase in heat stress by 3.0 degrees for 20 randomly sampled draws from the posterior of α_2.



- Pointwise posterior probability that $\alpha_2(\mathbf{s}) > 3.0$.
- Regions where all draws are greater than 3.0 (inside wide contour) or where no draws were greater than 3.0 (outside thin contour).



Varying thresholds ($\tau = 2.0, 3.0, 4.0$) for timeslice ($\boldsymbol{\alpha}_2$; top) and RCM ($\boldsymbol{\alpha}_2 + \boldsymbol{\alpha}_3$; bottom).



Pointwise probability for change greater than 3.0 for both models.

Questions?

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Thank You!